Radio frequency radiation beam pattern of lightning return strokes: A revisit to theoretical analysis

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[1] Return stroke current pulses can propagate at speeds approaching the speed of light *c*. Such a fast-moving pulse is expected to radiate differently than conventional dipole emitters. In this study, we revisit the theoretical analysis for the high-speed effect on the radiation beam pattern. Instead of starting with specific return stroke models, as has been done before by other investigators, we start the analysis with a general moving current pulse. Through a simple differential transformation between the retarded time and stationary time/space, the so-called *F* factor $(1 - v \cos \theta/c)^{-1}$ can be readily obtained. This factor is found to be fundamental and is explicitly associated with the radiation beam pattern but is not limited only to the lossless transmission line (TL) return stroke model. It is demonstrated that different beam pattern factors could be derived from this fundamental factor under different return stroke model assumptions. *INDEX TERMS:* 0619 Electromagnetics: Electromagnetic theory; 3304 Meteorology and Atmospheric Dynamics: Atmospheric electricity; 3324 Meteorology and Atmospheric Dynamics: Lightning; *KEYWORDS:* radiation beam pattern, return stroke, lightning

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1. Introduction

[2] Models that relate the observed electromagnetic (EM) field to the lightning current have been developed for return strokes, leader steps, and some general discharge processes, with the main attention focused on return strokes [e.g., *Uman and McLain*, 1969, 1970a, 1970b; *McLain and Uman*, 1971; *Uman et al.*, 1973, 1975; *Le Vine and Meneghini*, 1978a, 1978b; *Lin et al.*, 1980; *Meneghini*, 1984; *Rubinstein and Uman*, 1990; *Le Vine and Willett*, 1992; *Krider*, 1992; *Thottappillil et al.*, 1998; *Thottappillil et al.*, 2001].

[3] In terms of the far-field radiation, the early works by Uman and colleagues [e.g., *Uman et al.*, 1973] showed that the measured radiation intensity is directly proportional to the propagating current along the lightning channel if the discharge is assumed to follow a lossless transmission line (TL) model. To derive the explicit current-radiation relation for a TL model, most of their early studies assumed a vertical channel and a distant ground observer such that the viewing angle would be constantly $\pi/2$ and no other angular dependence was involved. In a general case without the far-field assumption, an apparent dipole beam pattern ($\sin \theta$) was found to be associated with the radiation term [e.g., *Uman et al.*, 1975].

[4] Assuming an arbitrarily oriented channel and by implementing the lossless TL model, *Le Vine and Willett* [1992] found that the previously inferred dipole field pattern [e.g., *Uman and McLain*, 1970a] needed to be "corrected" by a factor of $(1 - v\cos \theta/c)^{-1}$, or a "*F*" factor, to accommodate the effect of nonconstant retarded time along the channel segment. As pointed out by *Le Vine and Willett* [1992] and later by *Thottappillil et al.* [1998], the *F* factor was missed in the analysis of *Uman and McLain* [1970a, equation (7)] due to incomplete treatment of the varying retarded time, or equivalently, the varying lightning-observer distance.

[5] *Thottappillil et al.* [1998] reexamined the effects of the retarded time on the electric field by assuming an extending lightning channel instead of a preexistent, fixed channel segment as in *Le Vine and Willett* [1992]. The same *F* "correction" was obtained for the TL return stroke model. Nevertheless, for other return stroke models *Thottappillil et al.* [1998] came up with different "correction" factors. For instance, for a "traveling current source (TCS) model" [*Heidler*, 1986], a correction factor of $(1 + \cos \theta)^{-1}$ was found for the radiation pattern.

[6] In studies by *Le Vine and Willett* [1992] and *Thottappillil et al.* [1998] the analyses started from the integrated contribution along a presumed channel length, and the *F* factor was obtained through a mathematic identity involving the specific lossless TL current model. The former assumed a fixed channel length. The lightning current was injected into one end of the channel and was terminated or absorbed at the other end. The latter assumed an extending channel. For the TL model, the current initiated at the base of the channel and propagated forward without an apparent termination. This difference is the reason for the subtle discrepancy between equation (9a) of *Le Vine and Willett* [1992] and equation (38) of *Thottappillil et al.* [1998]. If the fixed segment were very long as

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compared to the scale of the current waveform, solutions for the TL model from the two reports would become identical.

[7] It should be noted that in the field of radio frequency antenna research, a similar problem has been studied for a traveling current wave along a wire antenna (physically identical to the TL return stroke model; see a review by *Smith* [2000]). The analytic result in the time domain is exactly the same as that reported by *Le Vine and Willett* [1992], except that for the antenna $v \equiv c$, and the corresponding F factor becomes $(1 - \cos\theta)^{-1}$. An antenna has a fixed length, the same as a fixed lightning channel segment as that assumed by *Le Vine and Willett* [1992].

[8] In this paper, we will revisit the theoretical analysis of the radiation beam pattern due to a traveling current pulse. The previous studies all started with a stationary coordinate frame attached to a presumed lightning channel (or an antenna) and derived the radiation component of the EM field based on stationary current variation ($\partial i(z, t - r/c)/\partial t|_{z=cont}$, z: spatial coordinate along the channel; t: time; r: distance between the channel element and the observer; and c: the speed of light). A stationary element of temporally varying current always produces a dipole radiation pattern and that is the reason for the sole $\sin\theta$ dependence in equations (7) and (9) in *Uman et al.* [1975a] and in other relevant papers. The F factor arises only for the simple TL model through line integration of the elementary dipoles, as discussed by *Thottappillil et al.* [1998].

[9] Return strokes and other lightning discharge processes can often be characterized by a traveling current pulse. In this paper, instead of treating the discharge solely in a stationary frame, we will start with a moving current pulse and will attach a moving frame to the pulse. To a stationary observer, it can be shown that an intrinsic, explicit (rather than implicit, model-dependent) F factor is needed for the radiation pattern. On the basis of this fundamental F factor, one could come up with rigorous, integrated correction factors for different lightning models.

2. Theoretical Analysis

[10] Figure 1 shows the problem under consideration, in which a current pulse propagates upward along a lightning channel and the radiation field is measured at point P(x, y, z). The instantaneous vector potential due to the current pulse ($i\,dz'$) is, in free space,

$$d\boldsymbol{A}(t, \boldsymbol{r}) = \frac{\mu_0}{4\pi} \frac{i(z', t')}{r(x, y, z, z')} dz' \hat{\boldsymbol{z}}, \tag{1}$$

where

$$t' = t - \frac{r(x, y, z, z')}{c} \tag{2}$$

is the retarded time. Here dz' is assumed moving along with the current pulse at speed v, rather than stationary in the observer's coordinate frame; z' is the instantaneous position of dz'; r is the distance from dz' to the observer; and $\hat{z}$ is the unit vector in the motion direction. The direction of $\hat{z}$ is arbitrary and needs not be limited to vertical.

[11] As indicated in equation (2), the retarded time t' is an implicit function of (x, y, z), in addition to being an explicit

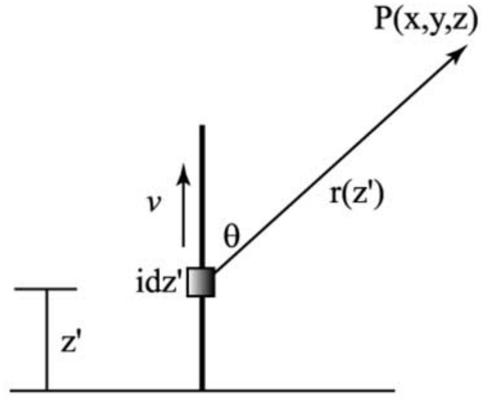

Figure 1. Geometry of the traveling current pulse and the observer.

function of t, so that any differential operation on (x, y, z) in the stationary frame would need to operate on t' too. This leads to

$$\begin{aligned} d\boldsymbol{B} &= \nabla \times (d\boldsymbol{A}) \\ &= \nabla \times (d\boldsymbol{A})|_{t'=const} + \nabla t' \times \frac{\partial(d\boldsymbol{A})}{\partial t'} \end{aligned} \tag{3}$$

In the (x, y, z) coordinates, t is an independent variable so that $\nabla t = 0$, but viewing from stationary P, $\nabla t'$ may not be zero, due to equation (2). We have [e.g., *Landau and Lifshitz*, 1962, p. 186]

$$\begin{aligned} \nabla t' &= -\frac{1}{c}\nabla r \\ &= -\frac{1}{c}\nabla r|_{t'=const} - \frac{1}{c}\frac{\partial r(t')}{\partial t'}\nabla t' \\ &= -\frac{\boldsymbol{r}}{cr} + \frac{\nabla t'}{c}\frac{(z - z')\frac{\partial z'}{\partial t'}}{\sqrt{x^2 + y^2 + (z - z')^2}} \\ &= -\frac{\boldsymbol{r}}{cr} + \frac{\nabla t'}{c}\frac{\boldsymbol{v}\cdot\boldsymbol{r}}{r} \end{aligned} \tag{4}$$

Rearranging the above equation, we have

$$\nabla t' = -\frac{\hat{\boldsymbol{r}}}{c(1 - \boldsymbol{v}\cdot\boldsymbol{r}/c)} \tag{5}$$

In equations (4) and (5), v is the instantaneous velocity of dz' at (z', t'). The physical interpretation of equation (5) is that the retarded time t' is not isotropic in the observer's coordinate frame.

[12] In equation (3), $d\boldsymbol{A} \propto 1/r$ and $\nabla \times d\boldsymbol{A}|_{t'=const} \propto 1/r^2$. For the radiation field, this term can be neglected as compared to the $1/r$ term. Substituting equation (5) into equation (3) we have

$$\begin{aligned} d\boldsymbol{B} &= -\frac{\hat{\boldsymbol{r}}}{c(1 - \boldsymbol{v}\cdot\boldsymbol{r}/c)} \times \frac{\mu_0}{4\pi}\frac{1}{r}\frac{\partial i(z', t')}{\partial t'} dz'\hat{\boldsymbol{z}} \\ &= \frac{\mu_0}{4\pi c}\frac{1}{r}\frac{\sin\theta}{(1 - v\cos\theta/c)}\frac{\partial i(z', t')}{\partial t'} dz'\hat{\boldsymbol{a}}_\phi \end{aligned} \tag{6}$$

Here θ is the angle from $\boldsymbol{v}$ to $\boldsymbol{r}$, and ϕ is the azimuthal angle around v. For a plane EM wave, which is justified for distant observations, we have

$$d\boldsymbol{E} = -c\hat{\boldsymbol{r}} \times d\boldsymbol{B} = \frac{1}{4\pi\varepsilon_0 c^2}\frac{1}{r}\frac{\sin\theta}{(1 - v\cos\theta/c)}\frac{\partial i(z', t')}{\partial t'}dz'\hat{\boldsymbol{a}}_\theta \quad (7)$$

[13] Equations (6) and (7) are general results for a traveling current pulse, which exhibit the F factor $(1 - v\cos\theta/c)^{-1}$ in addition to the classical dipole factor of sin θ. No special boundary condition, current distribution model were assumed. The observed radiation amplitude depends on the rate of current change at source z' at time t'. One should note that $\partial t' \neq \partial t$, and the physical current change rate $\partial i(z', t')/\partial t'$ should not be confused with the apparent current change rate $\partial i(z', t')/\partial t$ in the observer's frame.

[14] These general formulas are particularly useful if a current pulse changes its amplitude with time (e.g., attenuating along typical lightning channels) as it propagates. It should be noticed that in the case of a moving but unchanging current pulse, no radiation is generated. It should also be noticed that the F factor always applies as along as the current pulse moves.

[15] If a perfect conducting ground is considered and if the discharge is vertical and right on the surface of the ground, we have, by using equation (7) and following a similar analysis of *Krider* [1992]

$$d\boldsymbol{E} = \frac{2}{4\pi\varepsilon_0 c^2 r}\frac{\sin\theta}{\left(1 - (v\cos\theta/c)^2\right)}\frac{\partial i(z', t')}{\partial t'}dz'\hat{\boldsymbol{a}}_\theta \quad (8)$$

under the condition of $H'\cos\theta/c < 1/B$. Here H' is the height of the current pulse; B is the observing bandwidth, and $1/B$ is the ''intrinsic coherent time'' between the original and the reflected pulses [e.g., *Born and Wolf*, 1975, p. 319]. At VLF-LF, the coherent time is on the order of several μs to several hundred μs, corresponding to source heights of kilometers or higher, which is comparable to the actual scale of return stroke processes. In this case, equation (8), which is derived by adding a time-independent imaginary current source to the original, is valid at most of the times. With broadband VHF observation, the coherent time is shortened significantly. As will be discussed in a later paper (X.-M. Shao et al., manuscript in preparation, 2004), FORTE VHF receivers have a bandwidth of 22 MHz, and the corresponding coherent time is about 45 ns. In this case, if the source height is above ∼10 m, equation (8) will not apply anymore.

[16] On the basis of equations (7) and (8) the relative beam patterns for a vertical dipole, a traveling current pulse, both in free space, and a traveling current pulse just above the ground are illustrated in Figure 2. The current traveling speed v was assumed $0.75c$ for the corresponding cases. It is noticed that by using equations (7) and (8), the radiation intensity in all the three cases depends sorely on the physical current change rate $\partial i(z', t')/\partial t'$, and the beam pattern depends on an entirely separate parameter v. This makes the three different beam patterns directly compatible.

[17] As having been shown through the derivations from equations (3)–(6), the F factor is due to $\nabla t'$ and is caused by the nonlinear differential transformation between (z', t') and (x, y, z, t). In equation (3) it is clear that $\nabla t'$ only applies to the radiation term. *Kumar et al.* [1995] claimed that other components of the total electric field (static, induction) should also be ''corrected'' with a similar F factor; this is clearly wrong.

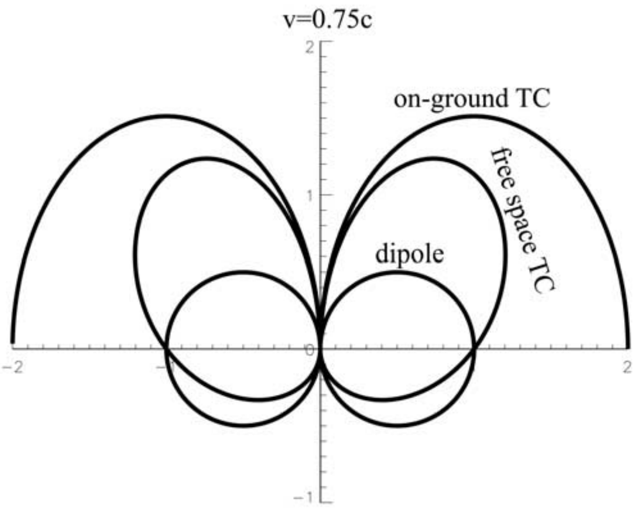

Figure 2. Theoretical beam patterns of radiation E field for (1) free space dipole (sin θ, equation (7), $v = 0$), (2) free space traveling current pulse (sin θ/[1 − v cos θ/c], equation (7)), and (3) on-ground traveling current pulse (2 sin θ/[1 − $(v\cos\theta/c)^2$], equation (8)). The speed v for the traveling current pulse is assumed $0.75c$.

3. Applied to TL and TCS Return Stroke Models

[18] For a linearly extended current source like a return stroke, equations (6) and (7) can be used as building blocks to derive the integrated radiation field, that is, in free space

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0 c^2}\int_0^{L'}\frac{1}{r}\frac{\sin\theta\hat{\boldsymbol{a}}_\theta}{(1 - v\cos\theta/c)}\frac{\partial i(z', t')}{\partial t'}dz' \quad (9)$$

Here $L'(=ut')$ is the physical length of the active channel measured at t', and u is the extension rate of the channel. This integration is mathematically similar to the previous studies that integrated over the stationary elementary dipoles, but is conceptually different that the entire channel L' can be considered as to have an instantaneous velocity of v, if v is the same for all the dz' s along L'.

[19] Under special conditions that (1) v is constant, (2) the channel is straight, and (3) the channel length is much shorter than the distance between the lightning and the observer ($L' \ll r$), the integration can be carried out as the following

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0 c^2 r}\frac{\sin\theta\hat{\boldsymbol{a}}_\theta}{(1 - v\cos\theta/c)}\int_0^{L'}\frac{\partial i(z', t')}{\partial t'}dz' = \frac{1}{4\pi\varepsilon_0 c^2 r}\frac{v\sin\theta\hat{\boldsymbol{a}}_\theta}{(1 - v\cos\theta/c)}(i(L', t') - i(0, t')) \quad (10)$$

One may notice that equation (10) is identical to equation (7) if L' is shortened to dz'. One may also notice that the fundamental beam pattern is the same as that in equation (7), and the other possible effects due to specific discharge models are entirely confined in a separate term, $i(L', t') - i(0, t')$. Again, this is conceptaully different than the previous studies in that the F factor was specificly based on the TL assumptions.

[20] In a TL model the channel extending at the same speed as the propagation of the current pulse, i.e., $i(L', t') = i(vt', t') = i(0, t' - vt'/v) \equiv i(0, 0)$. Assuming $i(0, 0) = 0$, the current at the channel base at $t' = 0$, equation (10) can be simplified as

$$\boldsymbol{E} = -\frac{1}{4\pi\varepsilon_0 c^2 r}\frac{v\sin\theta\hat{\boldsymbol{a}}_\theta}{(1 - v\cos\theta/c)}i(0, t') \quad (11)$$

[21] In equation (7), if one assumes the current pulse comes out from $z' = 0$ at speed v, but will not change its shape, amplitude (e.g., $\partial i(z', t')/\partial t') = 0$ when $z' > 0$) and speed thereafter, one may find that equation (7) becomes equation (11) through a transformation of $\partial i(0, t')/\partial t' dz' = v\partial i(0, t')$ and by removing the differential operators from the both sides. This is expected since these assumptions are the same as for a TL model. This further shows that the TL model is a special case for the general expression of equation (7).

[22] In a traveling current source (TCS) model [*Heidler*, 1986] the upward extending (at speed u) return stroke wave front instantaneously triggers current sources along the channel, and the triggered current propagates downward at the speed of $v = -c$, the speed of light. In this case, $L' = ut'$ and $i(L', t') = i(0, t' + L'/c) = i(0, L'(1/u + 1/c))$, and equation (10) becomes

$$\boldsymbol{E} = \frac{1}{4\pi\varepsilon_0 c^2 r}\frac{c\sin\theta\hat{\boldsymbol{a}}_\theta}{(1 + \cos\theta)}\left(i\left(0, L'\left(\frac{1}{u} + \frac{1}{c}\right)\right) - i(0, t')\right) \quad (12)$$

In this case, the F factor becomes $(1 + \cos\theta)^{-1}$ since $v = -c$.

[23] Derivations of equations (11) and (12) for the two simple return stroke models serve to illustrate the generality of equations (6) and (7) in terms of the F factor for a traveling current pulse. The F factor reported before for the TL return stroke model is apparently only a special case. For other more complicated return stroke models (e.g., with changing velocity, direction), one would need to start from the fundamental results of equations (6) or (7), and numerical integration might be required in these cases.

4. Conclusions

[24] From the theoretical analysis of this paper, it is clear that the F factor for the radiation field is an intrinsic, explicit factor over the dipole beam pattern, as long as a moving current pulse is considered. One could start from equations (6) or (7) to obtain specific radiation patterns for different return stroke models. The previously reported TL F factor is apparently only a special case. It is also clear that the F factor only applies to the radiation portion of the total field, but not to the static and induction fields.

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